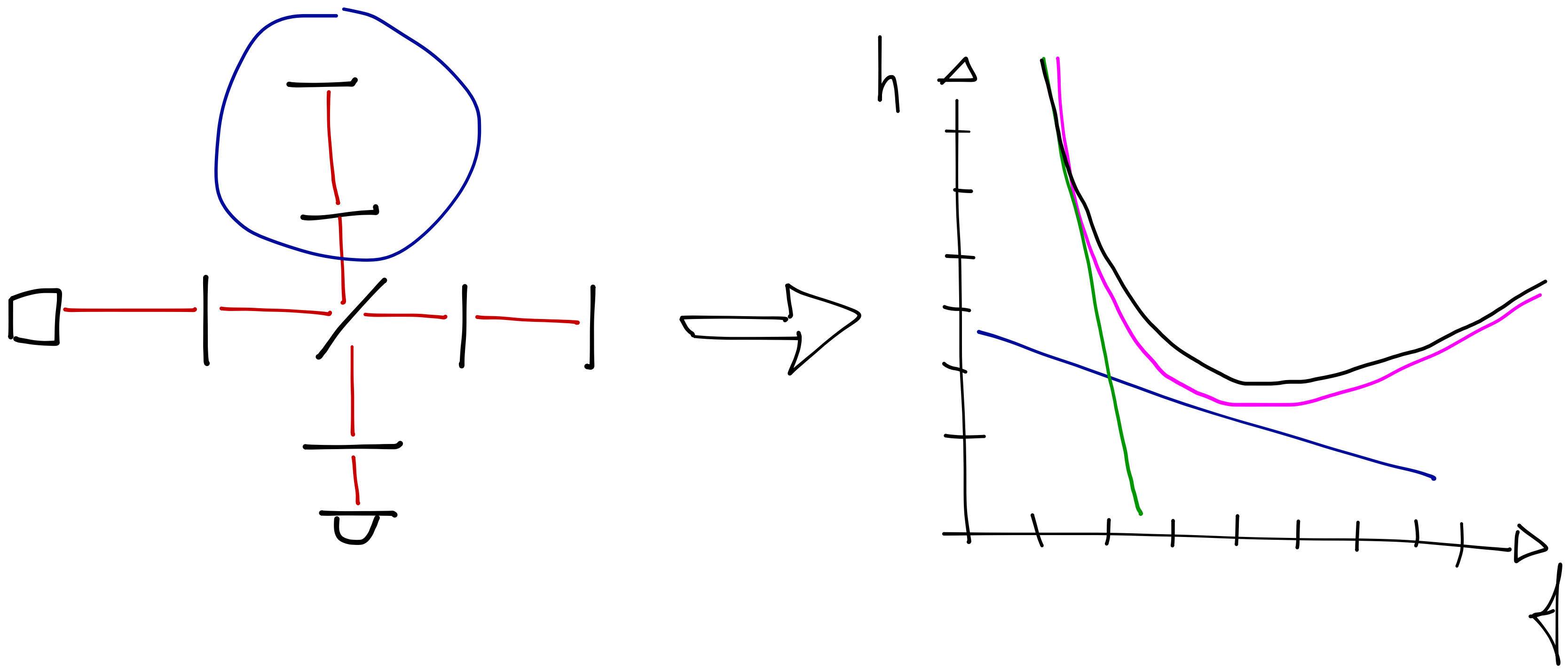
3 CAVITY PROPERTIES



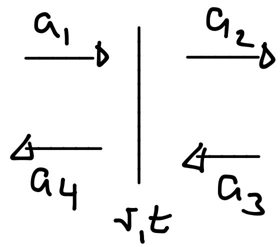
This session:

- properties of the optical cavity
- what are cavities used for in LIGO?

Yesterday, we went from

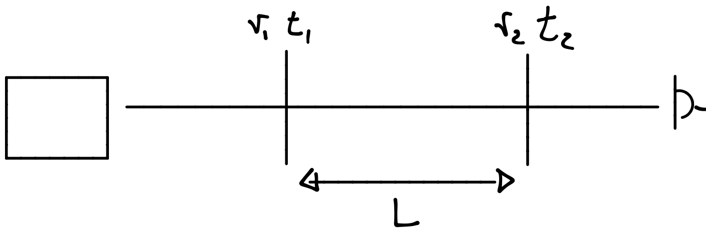
$$E = E_0 \, e^{i(\omega t - kz)}$$

and



$$a_2 = it a_1 + r a_3$$
$$a_4 = r a_1 + it a_3$$

to our first optical system



$$P = P_0 \frac{T_1 T_2}{1 + R_1 R_2 - r_1 r_2 \cos(2kL)}$$

What can we learn from this?

Let's look at particular examples!

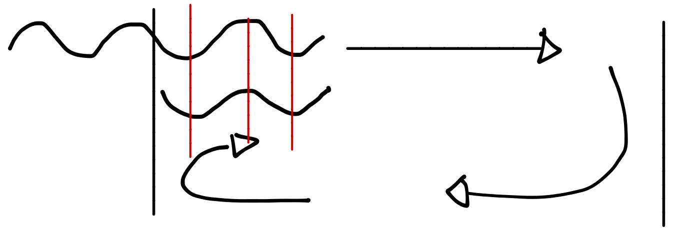
Assume a symmetric cavity: $T_1 = T_2 = T$, $R_1 = R_2 = R$

$$\rightarrow \quad P = P_0 \frac{T^2}{1 + R^2 - 2R\cos(2kL)}$$

$R + T = 1$

$T = 1 - R$

1. Resonance, when interference is constructive



$$\cos(2kL) = \cos(N \cdot \lambda \cdot k) = \cos(2\pi \cdot N) = 1$$

$$P = P_0 \frac{T^2}{1 + R^2 - 2R} = \frac{T^2}{(1-R)^2} = \frac{T^2}{T^2} = 1$$

On resonance this cavity transmits all the injected light.

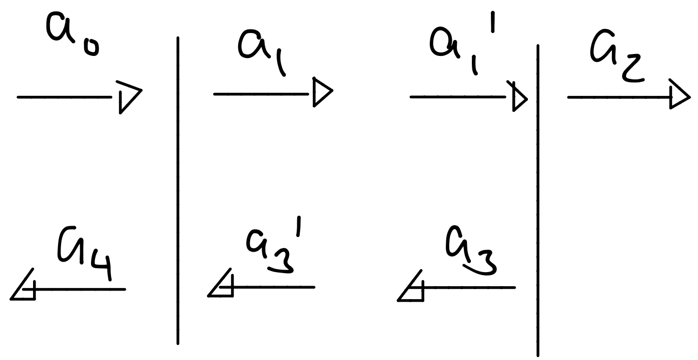
What can this be used for? A filter!

For $\omega = \omega_0 + \Delta\omega$, with $\omega = 2\pi \frac{c}{\lambda}$, $P = \begin{cases} 1 & \text{for } \Delta\omega = 0 \\ < 1 & \text{for } \Delta\omega \neq 0 \, ^{*} \end{cases}$

* and $\Delta\omega \neq 2\pi N \frac{c}{2L}$, see later

We found out that a cavity can be a spetral filter. This feature is helpful in LIGO but not the main purpose of the cavities.

Better to look at the field inside the cavity. (We'll show it once more in detail for practise.)



$$a_1 = it_1 a_0 + r_1 a_3' \qquad a_1' = a_1 e^{-ikL}$$

$$a_2 = it_2 a_1' \qquad a_3' = a_3 e^{-ikL}$$

$$a_3 = r_2 a_1'$$

$$a_4 = it_1 a_3' + r_1 a_0$$

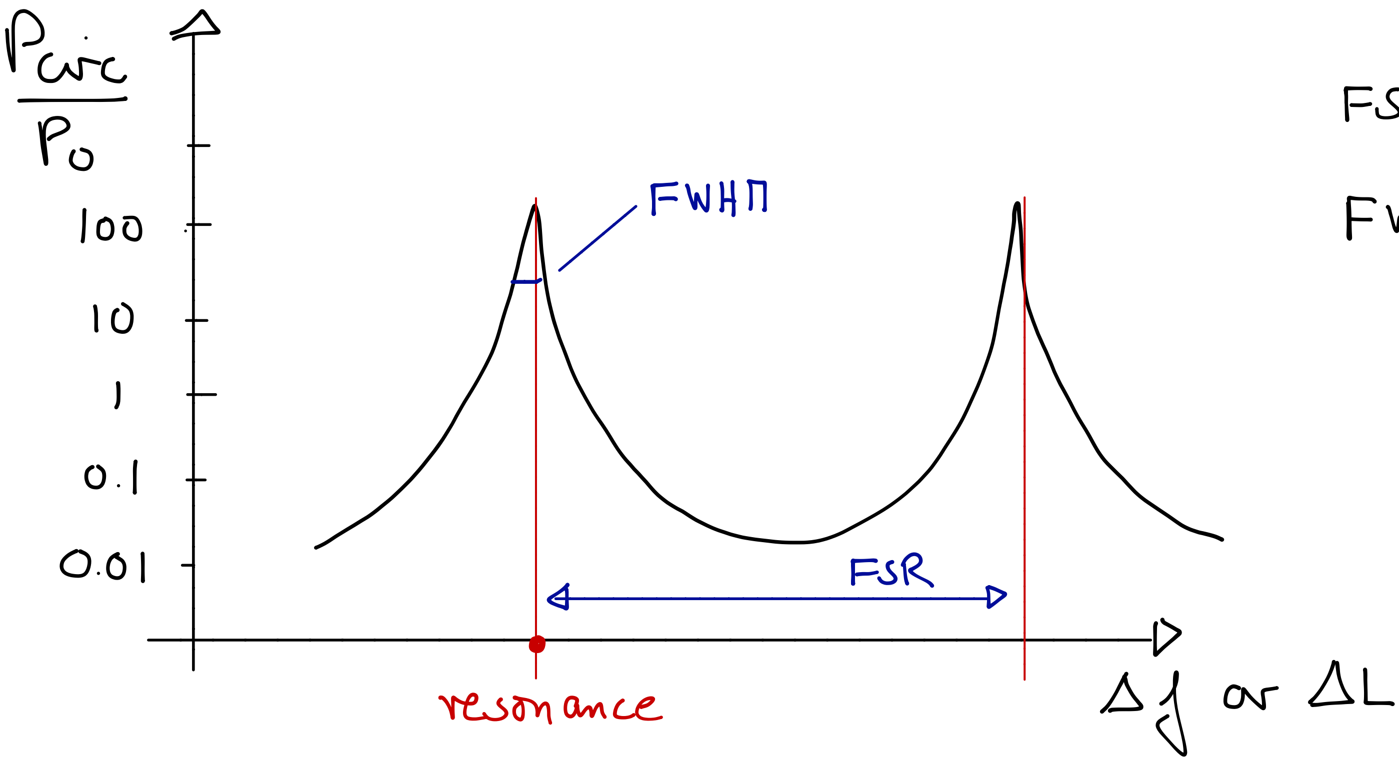
$$a_3' = r_2 a_1 e^{-2ikL} = it_1 r_2 e^{-2ikD} a_0 + r_1 r_2 e^{-2ikL} a_3'$$

$$a_3' = a_0 \frac{it_1 r_2}{1 - r_1 r_2 e^{-2ikL}} \longrightarrow \text{same as before!}$$

The factor $\frac{1}{1 - r_1 r_2 e^{-2ikL}}$ is the same for all cavity fields. It determines how the light power is affected by the factor 'kL' including the laser frequency ω and the cavity length L.

$$P_{circ} = P_0 \frac{T_1 \ (R_2)}{1 + R_1 R_2 - r_1 r_2 \cos(2kL)}$$

depending on the location in the cavity



FSR : Free Spetral Range

FWHM: Full Width at Half Maximum

$$FSR = \frac{c}{L_{\text{round trip}}} = \frac{c}{2L}$$ for a simple two-mirror cavity

FWHM: complicated equation

$$F = \frac{FSR}{FWHM}$$, the finesse, or quality factor of the cavity

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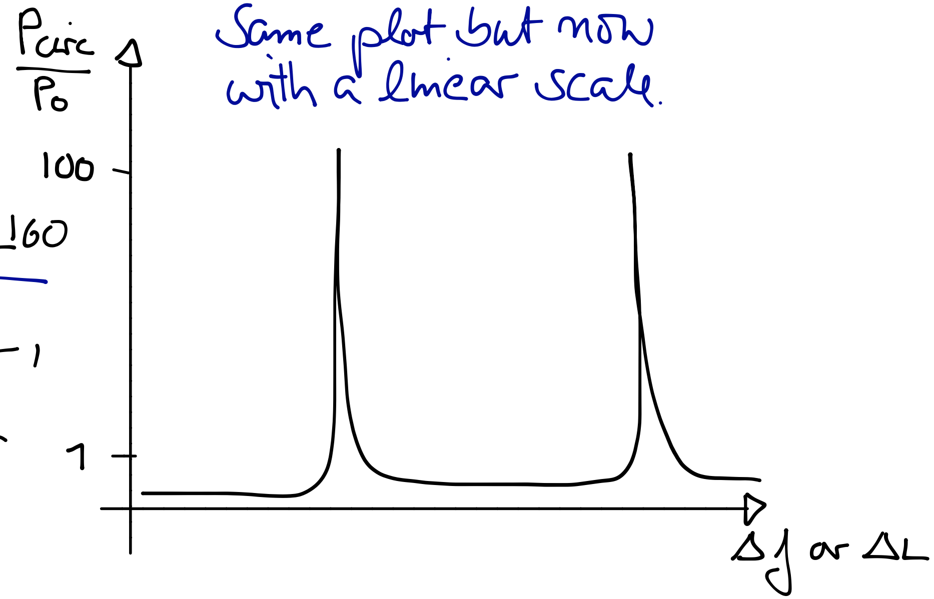
What can the cavity be used for?

1. filter light on transmission
2. power enhancement → main reason for cavities in LIGO
3. stabilisation: can be a reference for ω or L, in other words: can measure $\Delta\omega$ or ΔL very precisely.

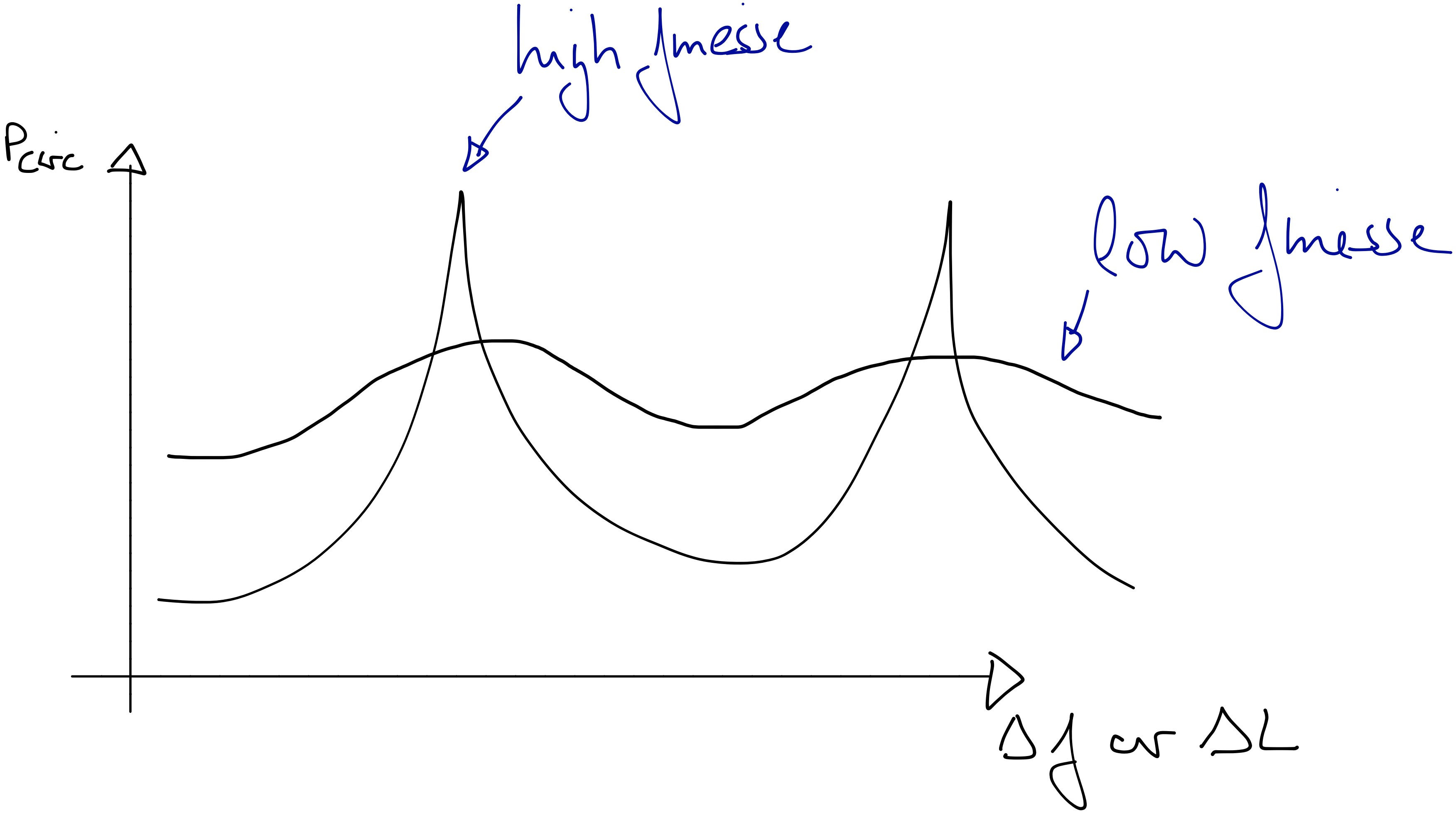
Same plot but now with a linear scale.



A few words on cavity parameters

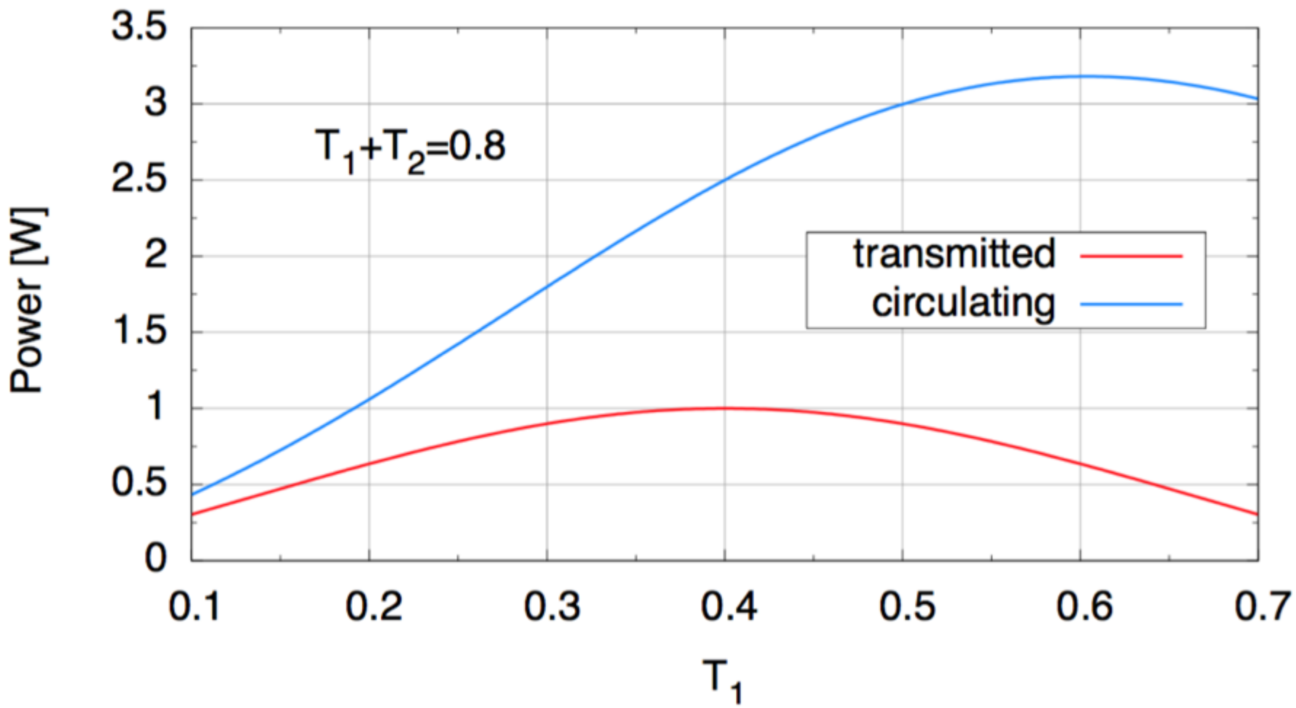
For high finesse (r_1 and r_2 close to 1) we can approximate

$$F \approx \frac{\pi\sqrt{r_1 r_2}}{1 - r_1 r_2} \approx \frac{\pi}{1 - r_1 r_2}$$



Special parameter ranges

$R_1 = R_2$, $T_1 = T_2$	$R_1 < R_2$, $T_1 > T_2$	$R_1 > R_2$, $T_1 < T_2$
Impedance matched, maximal throughput	Overcoupled, best power build-up.	Undercoupled



Same finesse for all points

L3

Impedance matched

$$F \approx \frac{\pi}{1-r_1 r_2} = \frac{\pi}{1-R} = \frac{\pi}{T}$$

Input	resonance	internal a_1	$\lvert a_1\rvert^2$	transmitted a_2	$\lvert a_2\rvert^2$	reflected a_4	$\lvert a_4\rvert^2$
external	resonant	$\frac{i}{\tau}$	$\frac{1}{T}$	-1	1	0	0
external	antiresonant	$\frac{i\tau}{2}$	$\frac{T}{4}$	$\frac{i\tau^2}{2}$	$\frac{T^2}{4}$	≈ 1	≈ 1
internal	resonant	$\frac{1}{\tau^2}$	$\frac{1}{T^2}$	$\frac{i}{\tau}$	$\frac{1}{T}$	$\frac{i}{\tau}$	$\frac{1}{T}$
internal	antiresonant	$-\frac{i}{2}$	$\frac{1}{4}$	$-\frac{i\tau}{2}$	$\frac{T}{4}$	$\frac{\tau}{2}$	$\frac{T}{4}$

Overcoupled

$$r_2 > r_1 \longrightarrow r_1 r_2 \approx r_1$$

$$r_1 = \sqrt{R} = \sqrt{1-T} \approx 1 - \frac{T}{2} \quad (T \text{ is small})$$

$$\longrightarrow F \approx \frac{\pi}{1-r_1 r_2} = \frac{\pi}{1-r_1} = \frac{2\pi}{T}$$

Input	resonance	internal a_1	$\lvert a_1\rvert^2$	reflected a_4	$\lvert a_4\rvert^2$
external	resonant	$\frac{2i}{\tau}$	$\frac{4}{T}$ *	-1	1
external	antiresonant	$\frac{i\tau}{2}$	$\frac{T}{4}$	1	1
internal	resonant	$\frac{2}{\tau^2}$	$\frac{4}{T^2}$	$\frac{2i}{\tau}$ **	$\frac{4}{T}$
internal	antiresonant	$\frac{i}{2}$	$\frac{1}{4}$	$\frac{\tau}{2}$	$\frac{T}{4}$

Usefull equations to remember

$FSR = \frac{c}{2L}$ → larger L makes smaller FSR

$F = \frac{FSR}{FWHM} \approx \frac{\pi}{1 - r_1 r_2}$, L does not affect the finesse

$\tau_{RT} = \frac{2L}{c} = \frac{1}{FSR}$ round-trip travel time for a photon

$\tau_c = \frac{1}{2\pi \, FWHM} = \frac{LF}{\pi c}$, average time a photon spends in the cavity

LIGO arm cavity

$L = 4\,km$

$R_2 \sim 1$

$R_1 = 0.986$, $T_1 = 0.014$

overcoupled

FSR = 37.5 kHz

$F \approx 450$ => FWHM $\approx 80\,Hz$

$\tau_{RT} \approx 26\,\mu s$

$\tau_c \approx 2\,ms$

power enhancement ≈ 300

Summary:

- cavity fully described by L, R_1, R_2 (T_1, T_2)
- derived useful parameters of the optical resonators FSR, FWHM, F

Next: Michelson interferometer